To be and to belong

The definition of a set is always arbitrary. That fact gives them the double character of *unjustifiable* and *unquestionable*. If a definition does not give rise to more than one interpretation, it cannot be objected. That would require the fulfillment of two conditions: (1) that each element in the universe is clearly within or without the set; and (2) that the name of the set has not been used for another one with a non-equivalent definition.

Arbitrariness is a characteristic of the definitions for entities of every kind, including mathematical ones. Let us consider the aforementioned case of the natural divisors of 12. If the meaning for "natural divisor" were not clear, two interpretations would be possible.

$$A_1 = \{1, 2, 3, 4, 6, 12\}$$

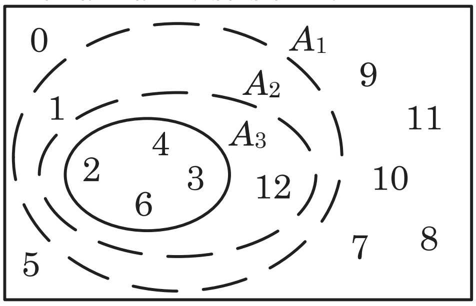
$$A_2 = \{2, 3, 4, 6, 12\}$$

Each enumeration correspond respectively to the following definitions: (A1) *Natural numbers that divide 12 and leave no remainder;* (A2) *Natural numbers that reduce 12.* The second definition may seem whimsical, but not if you look at the etymology of the word "divide," because the act of dividing should always arrive at a quotient that is smaller than the dividend. Dividing by 1 is like adding 0: strictly speaking, neither is either dividing or adding. On the other hand, using the same name for either set would invalidate both definitions; therefore different symbols are used, A_1 and A_2. Giving an unequivocal definition to the divisors of 12; one that meets both of the mentioned conditions, is an ontological task. Both definitions are valid. Arguments could be given in favor of one or the other, but that is not the purpose of ontology.

When comparing the two definitions in the previous paragraph, yet another definition for "divisor of 12" becomes apparent. In fact, in definition A_1, the numbers that result from dividing 12 by each element of the set also belong to the set: 12/1 = 12, 12/2 = 6, 12/3 = 4, 12/4 = 3, 12/6 = 2, and 12/12 =1. In definition A_2 this is no longer true since the element 1 has been eliminated, which makes the definition less symmetric. The solution would then be to eliminate the element 12 too.

$$A_3 = \{2, 3, 4, 6\}$$

Thus arriving at the following definition: (A_3) *Natural numbers that reduce 12 such that the results also belong to the set.* The following diagram illustrates the definition variants for "the natural divisors of 12."



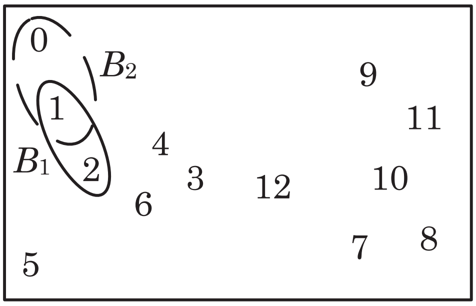
ontology

Now consider the following definition: The first two natural numbers. This definition does not answer whether or not zero should be considered a "natural number."

$$B_1 = \{1, 2\}$$

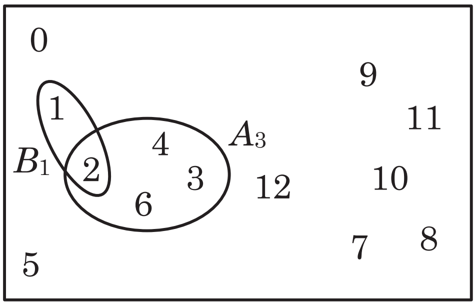
$$B_2 = \{0, 1\}$$

The following diagram illustrates the definition variants for "the first two natural numbers."



ontology

Once the definitions have been made—the field of ontology—the relative position of the sets—the field of dialectics—can be studied.



dialectics

The dialectic game begins when a universe contains more than one definition. Ontology makes many definitions, but does so one by one, without differentiating them from one another and therefore not justifying them.

Once the definitions are chosen (those with unbroken lines) and brought together in the same universe, the sets can be used in operations. And thus we enter into the field of logic, the *third philosophy*. For example, we could ask what elements of set A_3 do not belong to B_1. To answer this question, the elements that are common to A_3 and $\sim B_1$ must be found, that is, the elements resulting from

(continued on page 3)

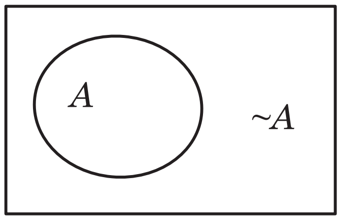
FRONT PAGE

Glossary of Ontology

(continued from page 1)

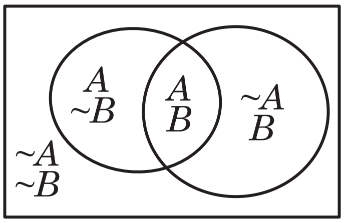
Ontology is closely related to semiology, which studies the relationship between signs and their meanings. The latter goes as far as not distinguishing between concrete and abstract entities. They all show signals that we can perceive with our senses. A set could contain a brick and a number, provided that a definition has placed them there. With ontology, the following question could be raised: What comes first, the being or the definition? However, this question is pointless because: an entity, in the sense **2b** does not become an entity in the sense **2a** until a definition is applied; and a definition is not possible without a universe and elements with shared characteristics.

Ontological tables: Are tables that can serve the function of Venn diagrams. In these tables, the elements of a set are on the same column or row while on the diagrams they are within an enclosing line. In both cases, no elements can be placed over the lines and the names of the sets are used as labels. The following are the diagram and table representations used in the case of a universe with only one definition, *A*.



A	*~A*

In a universe with two definitions, A and *B*, the following representations are used.



	A *B*	*~A* *B*
	A *~B*	*~A* *~B*

Tables with more than one definition are known as *Carroll diagrams,* named after the English writer, mathematician and logician Lewis Carroll (1832-1898).

The three forks
on the road of being

The verb "to be" has two senses. Consequently raising two issues (that of existing and that of being), to which a dilemma is added (that of belonging), that comes out from the limits of the universe.

The *dilemma of belonging* arises when a definition is made within a universe and it can be summarized with the following statement: *An element in the universe either belongs to a set or to its complement.* This dilemma appears when the elements can be named. All of the elements in the universe are something, that is, they are in the sense **1a** (see the front page article). This situation is summarized in the following table.

dilemma of belonging (Carroll)	
A	$\sim A = \mathcal{U} - A$
to be-something in a direct way	to be-something in an indirect way
to belong to the set	to belong to its complement

The universe is the part of the whole that is being considered.

$\mathcal{U}$	*The Whole*
to be-something	to be
to be in a real sense	to be in an abstract sense
to belong	to exist

The elements of the universe appear, while elements outside of the universe hide. Some of them can quickly be brought into the universe, others take more effort and yet others—it might be thought—shall never enter. But all of these elements have something in common: they simply exist, they have no name. The situation is presented in the following table.

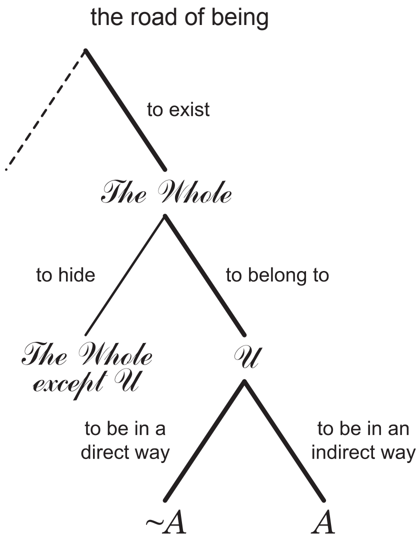
the issue of being (Parmenides)	
$\mathcal{U}$	*The Whole except* $\mathcal{U}$
to be-something	to be but not to be-something
to be in real sense	to be in abstract sense but not to be in real sense
to belong	to exist but not to belong
to have name	to exist but not to have name
to appear	to hide
not to hide	not to appear
ἔστιν	οὐκ ἔστιν

The issue of being, which can be stated like this: *An element of the whole is contained in the universe and is something* (in the sense **1a**) *or it is outside of it and simply is* (in the sense **1b**), lies between these two worlds.

Finally, all elements exist: *"The entity is."* If they ceased to exist, they would cease being elements. They would be non-existent, in the sense that there is no way to name that which does not exist, other than the roundabout method of saying "that which does not exist." Nonexistence is not an option for an entity; it would mean its death. There is no conservation law for entities. The *issue of existing* is shown in the following table.

issue of existing (Xenophanes, Hamlet)	
The Whole	
that which exists	that which not exists
that which is in abstract sense	that which is not in abstract sense
ὡς ἔστιν	

In summary, there are three forks on *the road of being.* The first one is the existential issue, of interest to literature and religion; the second is the essential issue, of interest to philosophy; the last one, the dilemma of belonging, is of interest to mathematics and science.



This image of the road of being solves one of the enigmas in Parmenides' poem. The first paragraph, which was printed in the same space of the previous issue, Parmenides refers to the *first fork.* There he speaks of *being* in an abstract sense (**2b**), ὡς ἔστιν = *that which is,* and calls it "whole" and "homogenous," because it is the only one, another road does not exist. In the second paragraph, he refers to the issue of being, that is, to the *second fork.* There he speaks of *hiding,* οὐκ ἔστιν, i.e. what is done by something that cannot be named, as opposed to *appear,* ἔστιν, i.e. what is done by something that can be called "true" (being, in the sense **2a**).

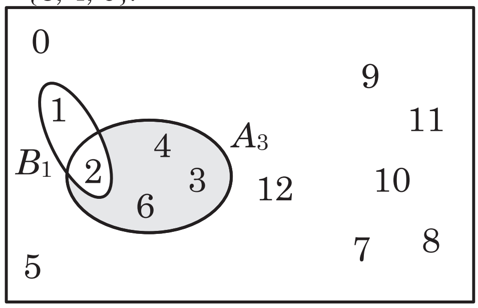
Translating a work like Parmenides', of which we only have fragments, far removed in time, involving concepts that are specific to a language and acquired while learning to speak, and written in Homeric Greek, is a hard task. The three forks mentioned here are related to three great questions, but that shall be the subject of a future article.

MAIN ARTICLE

To be and to belong

(continued from page 2)

the operation $A_3 \cap \sim B_1$. Each term in this expression corresponds to an affirmation: first, the affirmation "the element belongs to A_3;" second, the affirmation "the element belongs to $\sim B_1$." The result is the region where both affirmations are valid: $A_3 \cap \sim B_1 = \{3, 4, 6\}$.



logic

Logic is that part of philosophy which links affirmations to reach new affirmations. The conclusion drawn from the previous example is: *To simultaneously be two things something must be each one of them separately.* In set theory terms: *to belong to two sets is to belong to each one of them.*

Ask Jotajota

Send your question to: jjluetich@luventicus.org

Francisco from Monterrey (MX) asks:

—*¿What is the difference between criterion and definition?*

—To define a set is to group together the elements of a universe. This grouping may be achieved without applying any criterion (for example, choosing elements at random), applying a criterion that cannot be formalized (for example, of an aesthetic kind), or applying a criterion that can be formalized (for example, of a mathematical kind). In the first case, the definition can only be made by enumerating the elements; in the second case, the subject that applies that criterion must be consulted to decide whether or not an element belongs to the set; in the last case any selector would reach the same result. This is the most interesting one, not only because the definition is independent of the subject (or common to many subjects) but because concepts are derived from that sort of definition. The concept of sports, for example, arises from a definition made with a formally expressed criterion, which in turn is the result of analyzing a series of specific cases (elements). Confronting definitions of this kind is the subject of dialectics.

José Antonio from Guayana City (VE) asks:

—*¿Why isn't three the same as 3?*

—The first sign, according to the context, may correspond to an indeterminate number, as when you say "some three persons." The second is the formal (mathematical) expression for an amount. A set must not have identical elements, or different names for the same element.